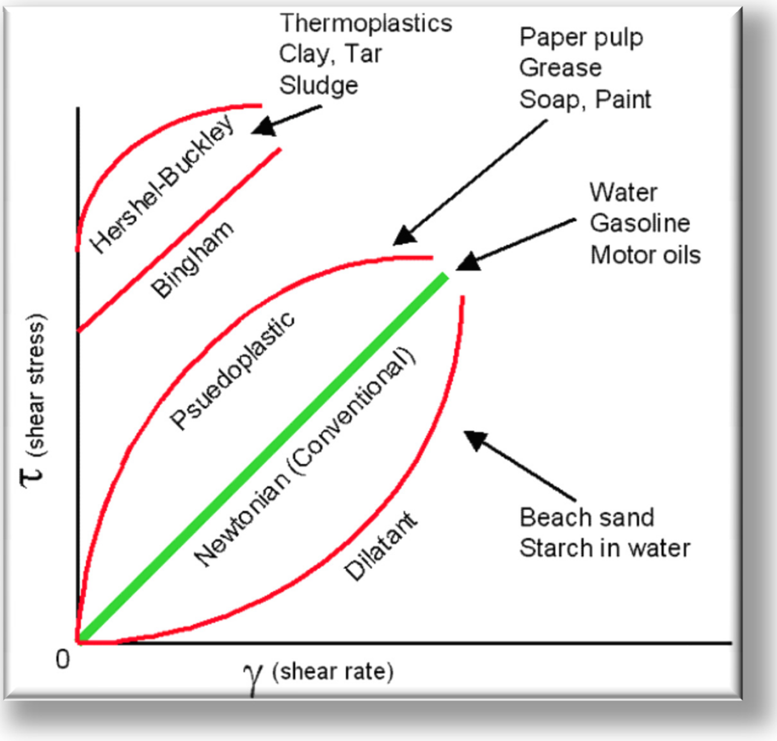
Environment and Computer Laboratory HS09
Water Resource Management

– Exercise 2 –

Fluid Dynamics | Non-Newtonian Fluids



– Report –

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Content

Introduction

The aim of this exercise is to understand the behavior of non Newtonian fluids. Non Newtonian fluids are important fluids in our daily life, e.g. toothpaste, oils, foods and so on. With a given MATLAB-Code a time independent 1D dam break-like slumps will be simulated for different fluid parameter like the fluid yield stress, the flow index and the consistency index. Figure 1 shows the experiment for the slump.

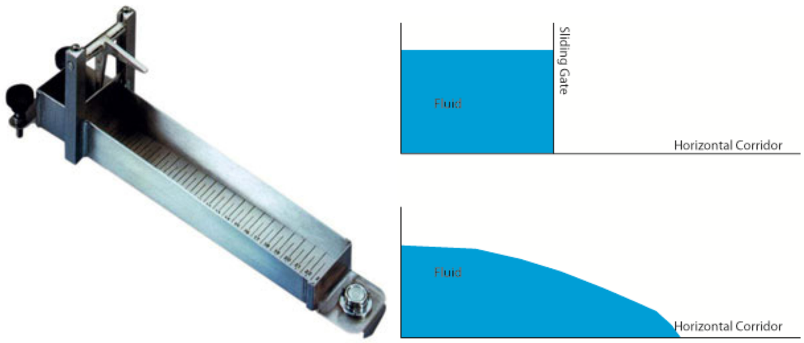


Figure 1: Bostwick Consistometer (left), description of the experiment (right)

Task 1 | Effects of the fluid yield stress

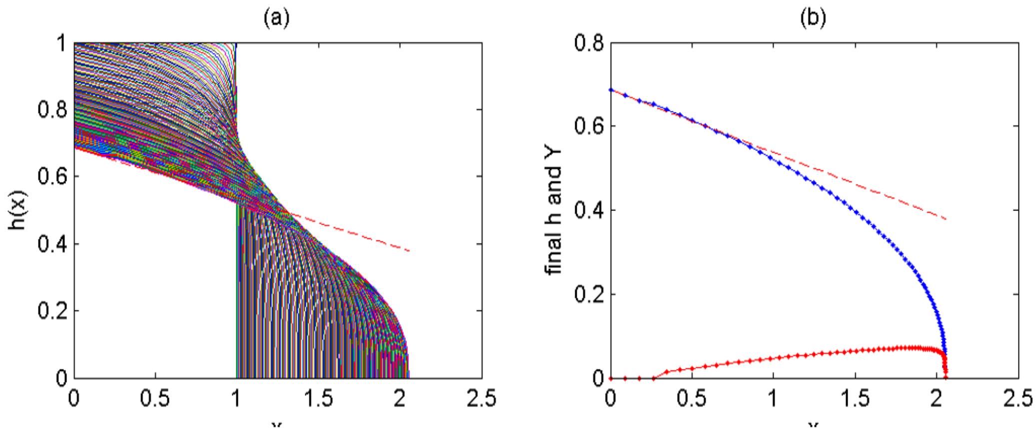
The Bingham Number B (see Equation 1) is a dimensionless number in which all the effects of the fluid yield stress are summarized. For ten different Bingham numbers in the range between 0 and 1 the fluid front evolution is plotted with the given MATLAB-Code. The flow index n is keeping constant at a value of n=2.

$$B = \frac{\tau_y \cdot L}{\rho \cdot g \cdot H^2} \tag{1}$$

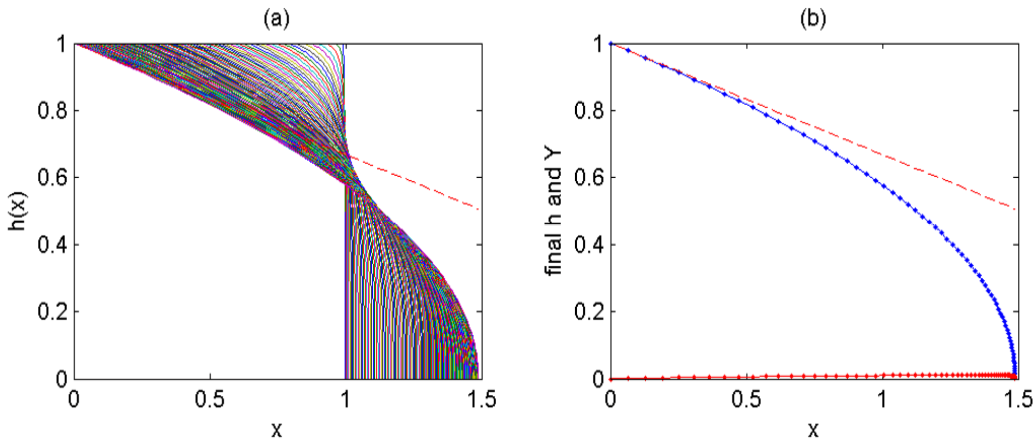
The results are presented in Figure 2. In this figure one can see the front evolution for B=0.10, B=0.33 and B=1.00. For B<1/3 one can see that the evolution front moves. The fluid moves forward from x=1 to x=2.5 and the height of the evolution front decreases. For B=0.33 the front stops at x=1.5. A particle at position x=0 of the front does not move. The B=1.0 case shows that no particle moves at position between x=0 and x=0.2. The end of the front is at approximately x=1.2. More figures for other Bingham Numbers can be found in the appendix.

The Bingham number describes the fluid yield stress. For high Bingham numbers B the evolution front stops earlier. To reach the same position it needs more force so that the fluid moves.

B = 0.10



B = 0.33



B = 1.00

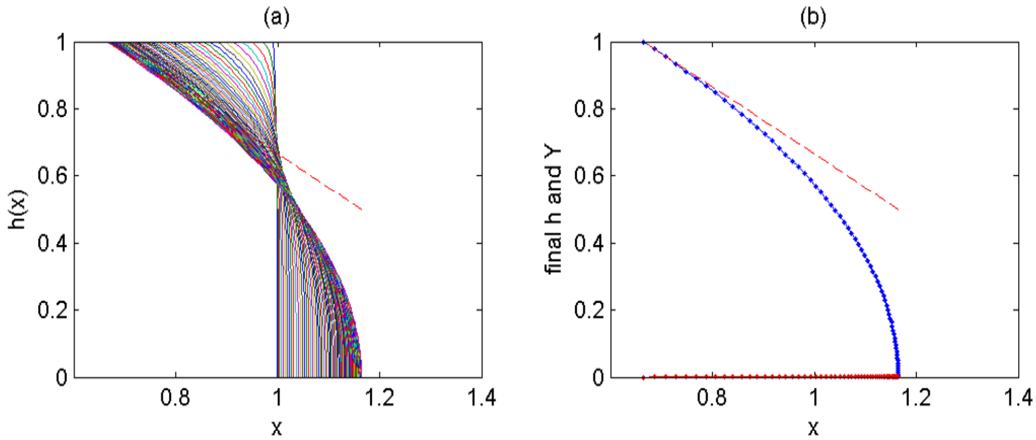


Figure 2: Evolution of the front position and its height (left) and end position of the fluid (right) for different Bingham numbers between B=0.1 and B=1.0

Task 2 | Effect of the flow index n

The flow index n shows is the parameter that describes how "Non-Newtonian" a fluid is. A Newtonian fluid has a value for n=1, i.e. the shear stress is proportional to the shear rate. For Non-Newtonian fluids the shear stress is a power function of the shear rate (see Equation 2).

$$\tau = -K \cdot \dot{\gamma}^n \qquad (2)$$

For two constant Bingham numbers (B = 0.1 and B = 0.6) the flow index n was varied between n=0.2 and n=3.0. The results are plotted in Figure 3.

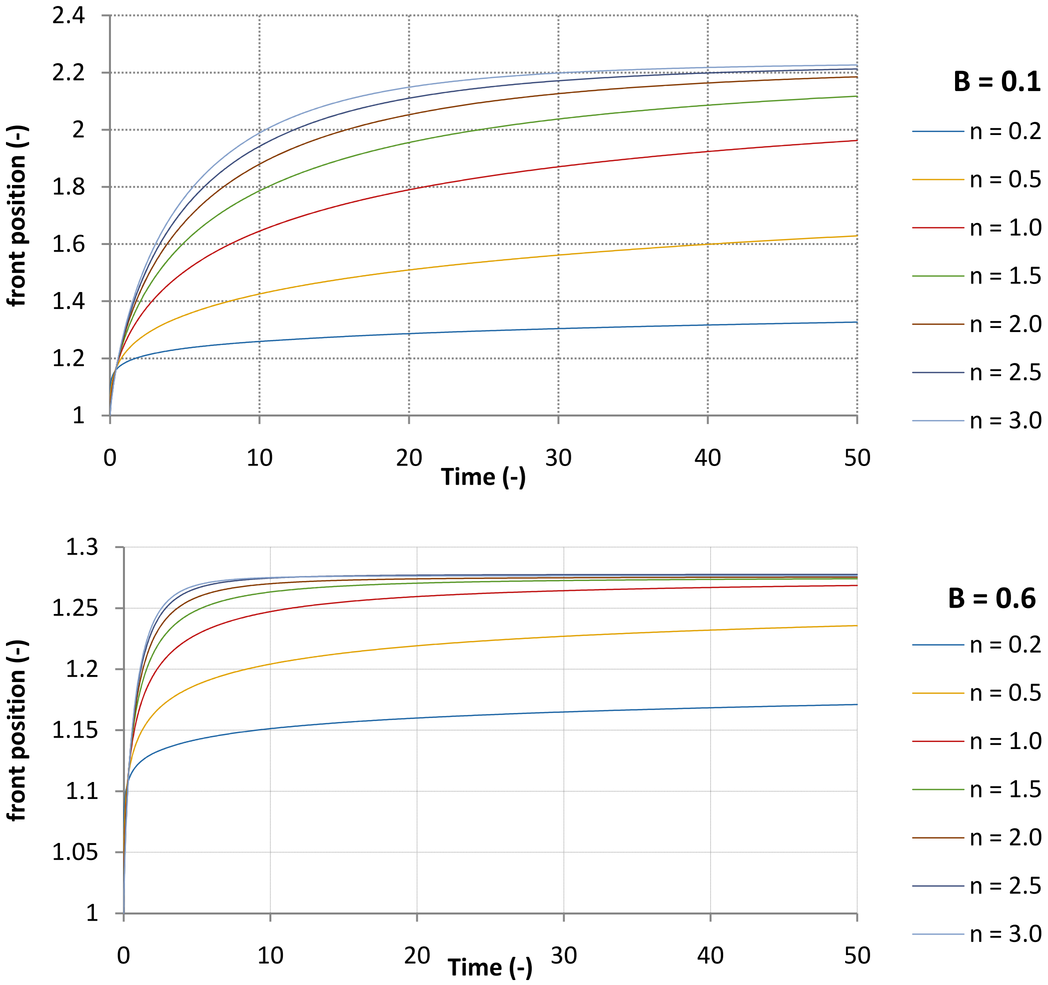


Figure 3: Evolution of the front position for two Bingham numbers (B=0.1: top, B=0.6 down) for different flow indexes between n=0.2 and n=3.

For B=0.1 one can see, that for $n > 1$ the curve begins to converge to 2.2 front position units after around 30 and 50 time units. For $n < 1$ the front position did not increase much. For n=0.2 the curve convert to 1.4 front position units.

For B=0.6 the convergence is much faster, but the front position unit is lower. For $n \geq 1$ the front position converts to 1.27 units. For n<1 the curve reaches its maximum faster. For n=0.2 the curve grows slowly after around 10 time units.

For a low Bingham number (B=0.1) the front position varies for different flow indexes between 1.3 and 2.2 front position units. For Bingham number B=0.6 the variation is much smaller and lies between 1.17 and 1.27 front position units. Bingham number and fluid index number are important to know, if the Bingham number is low. For Bingham number B>0.33 the flow index *n* is only for n<1 important. For fluids with an index n>1 one can approximate with n=1.

Task 3 | Real coordinate system

The results out of the MATLAB program can be converted in a real coordinate system. The conversion to this coordinate system can be done with the following equations:

$$x = L \cdot x \tag{3}$$

$$h = H \cdot \hat{h} \tag{4}$$

$$z = L \cdot \hat{z} \tag{5}$$

$$t = \frac{L}{H} \cdot \left(\frac{K \cdot L}{\rho \cdot g \cdot H^2} \right)^{\frac{1}{n}} \cdot \hat{t} \tag{6}$$

For a constant Bingham number B=0.6, two different flow index number n=0.2 and n=3 and for the consistency index $K = 10$ kg m^{-1} s^{n-2} and $K = 100$ kg m^{-1} s^{n-2} the results of this conversion are plotted in Figure 4.

For n=0.2 the curves look equal for $K = 10$ kg m^{-1} s^{n-2} and $K = 100$ kg m^{-1} s^{n-2}. The front position reaches the same point. Only the time is different. For the $K = 10$ kg m^{-1} s^{n-2} case it takes much less time (around 2.25E-12 seconds) to reach the end position than for $K = 100$ kg m^{-1} s^{n-2} case with around 2.25E-7 seconds.

For n=3 and $K = 10$ kg m^{-1} s^{n-2} it takes around 2.5 second to reach the end position while for $K = 100$ kg m^{-1} s^{n-2} it takes around 5 seconds.

The results for other *B* and *n* are shown in Figure 6 in the appendix.

K has only an effect on the time. It follows that a fluid with a high consistency but same Bingham and index flow number needs more time to reach the end position. The shape and end position remain the same though.

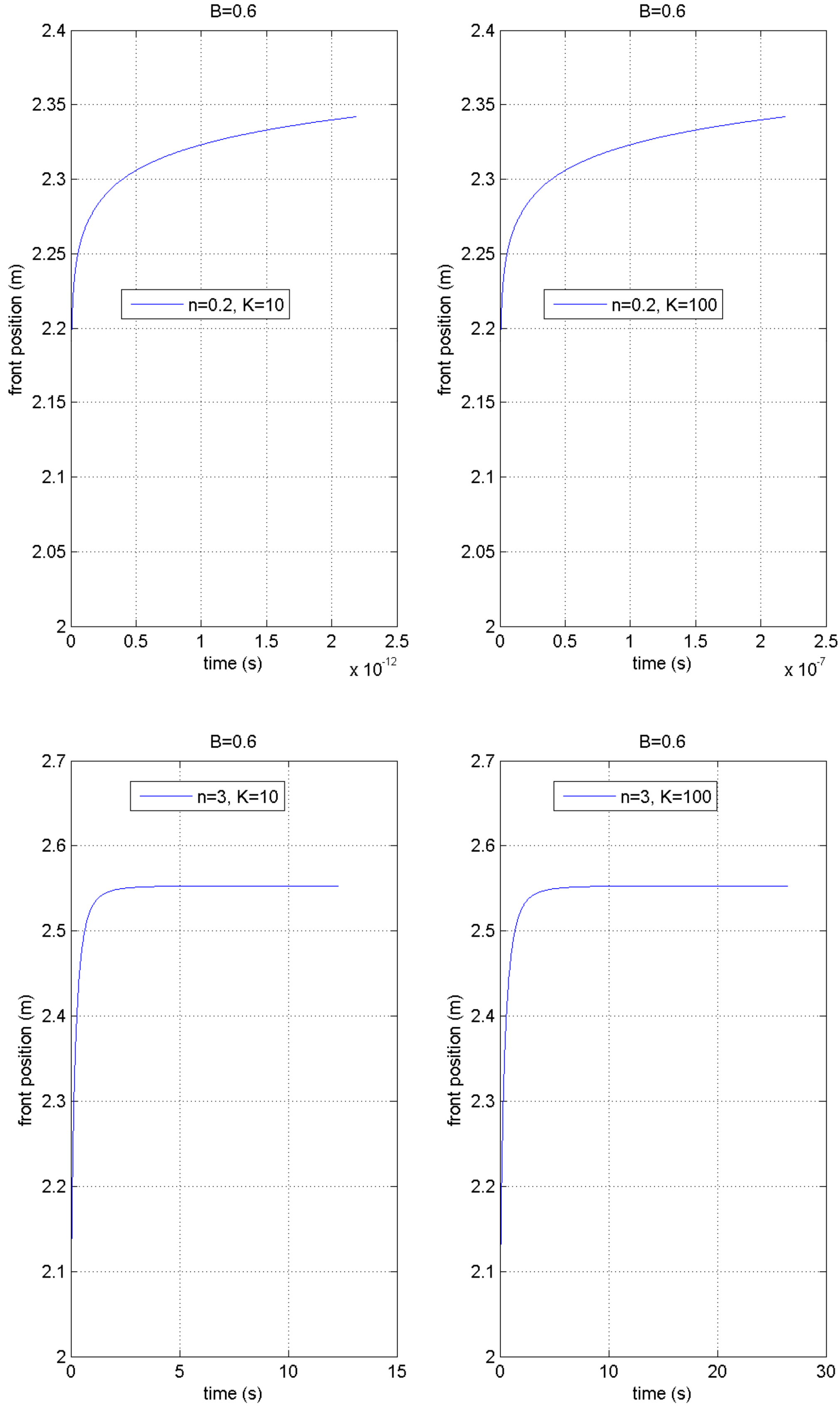


Figure 4: Evolution of the front position for two different flow indexes n (n=0.2: top, n=3: down) and two different consistency index K (K=10: left, K=100: right).

Appendix

Task 1

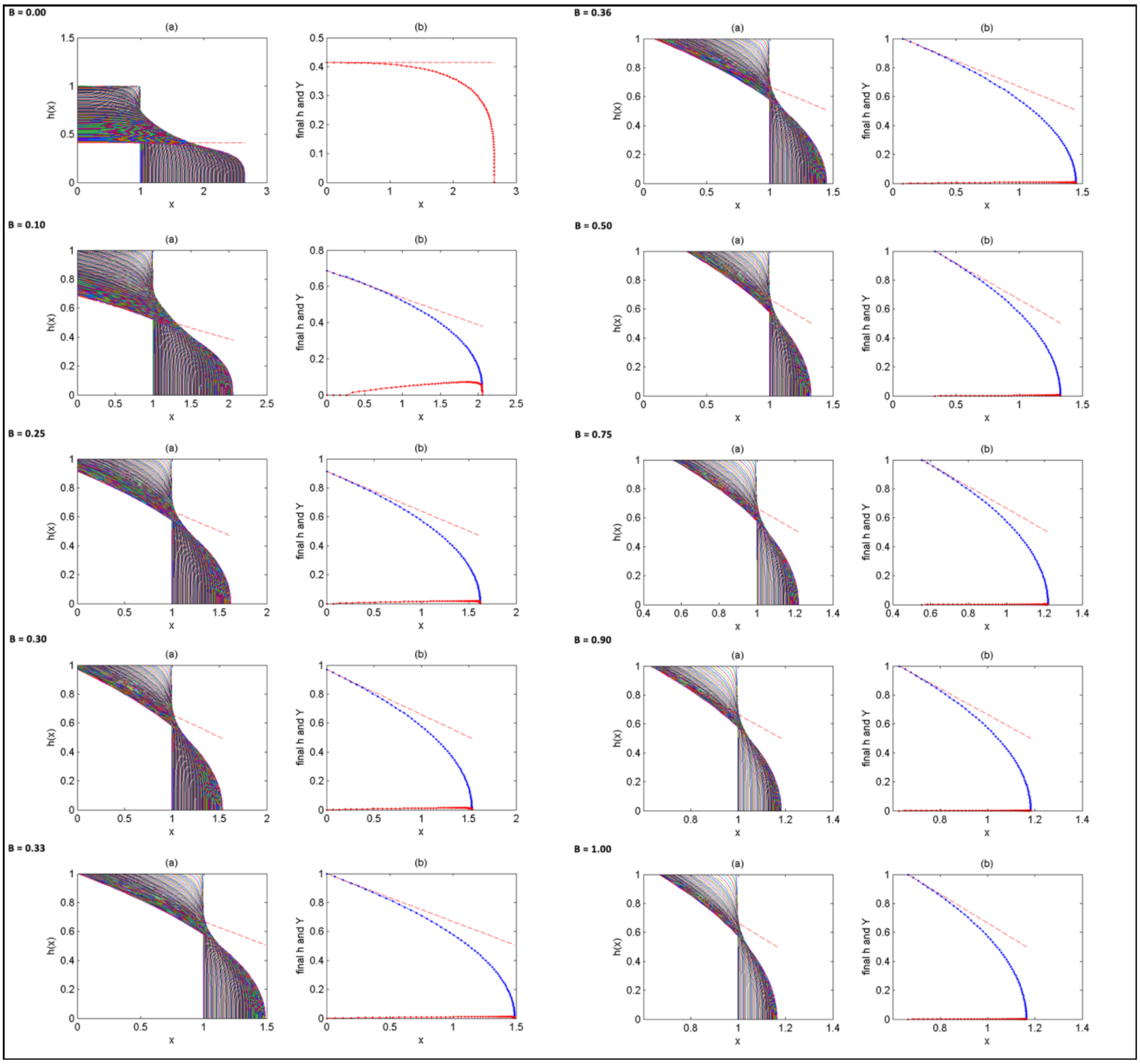


Figure 5: Evaluation of the front position and its height (left) and end position of the fluid (right) for different Bingham numbers between B=0.1 and B=1.0

Task 3

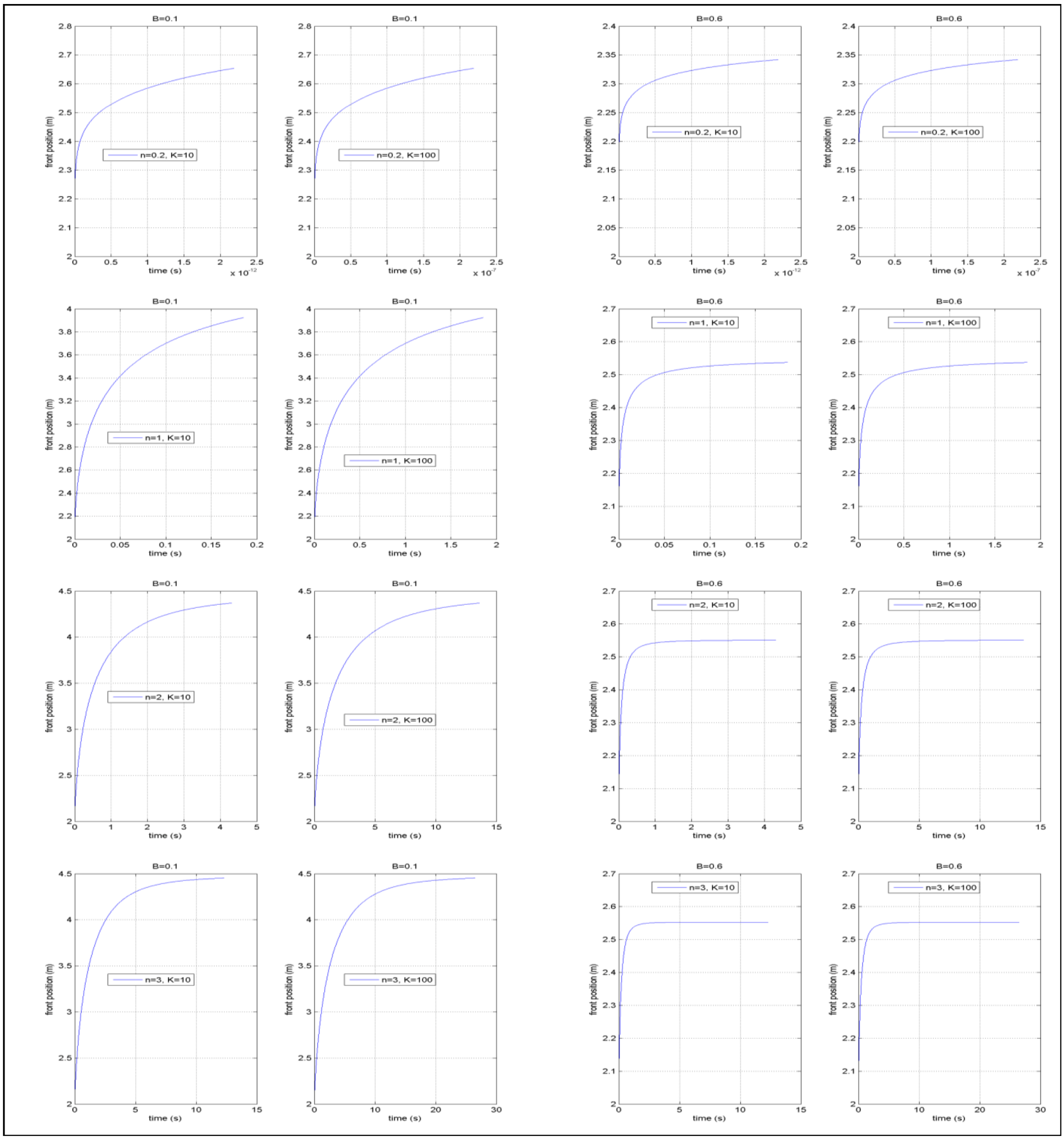


Figure 6: Evolution of the front position for two different Bingham numbers B (B=0.1 and B=0.6) and two different consistency indexes K (K=10 and K=100). The flow indexes n varies between n=0.2 and n=3.